Using Schema-Based Instruction to Improve Students' Learning of Mathematical Word Problems

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Presentation Agenda

- Site roll call and Math Cue Card Summary
- Introduction to Schema-Based Instruction (SBI)
  - Overview of Research on Mathematics Instruction for Students with Disabilities
  - What is Schema-Based Instruction (SBI)?
  - SBI: Application to Change Problems
  - SBI: Application to Group Problems
  - SBI: Application to Compare Problems
  - SBI: Application to Two-Step Problems
  - Questions and Answers
- Complete evaluations, give to room host, and collect attendance certificate

Mathematical Proficiency

1. *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations
2. *Procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. *Strategic competence* – ability to formulate, represent, and solve mathematical problems
4. *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
5. *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

National Research Council (2001, p. 5)

Research on Effective Mathematical Practices For Students With Learning Difficulties

- A summary of major findings from three syntheses of research on effective practices for students with mathematics difficulties, including over 50 studies, serve as the basis for developing interventions for students who require more than what typical classrooms can provide.
- These practices benefit students who:
  - enter school with very limited knowledge of number concepts and counting procedures
  - receive inadequate instruction in previous years of schooling and fall behind their peers
  - still continue to experience problems regardless of motivation, quality of former mathematics instruction, and number knowledge and number sense when entering school

Gersten, Baker, & Chard (2006)

Effect Sizes for Instructional Variables

Instructional Strategy	Effect Size For Special Education Students	Effect Size For Low Achieving Students
Visual and Graphic Depictions	**.50** *Moderate*	NA
Systematic and Explicit Instruction	**1.19** *Large*	**.58** *Moderate to Large*
Student Think Alouds	**.98** *Large*	NA
Use of structured peer-assisted learning activities involving heterogeneous-ability groupings	**.42** *Moderate.*	**.62** *Large*
Formative Assessment Data Provided to *Teachers*	**.32** *Small to Moderate*	**.51** *Moderate to Large*
Formative Assessment Data Provided Directly to *Students*	**.33** *Small to Moderate*	**.57** *Moderate to Large*

Summary

Results of these research syntheses suggest that students who are struggling with mathematics benefit from:

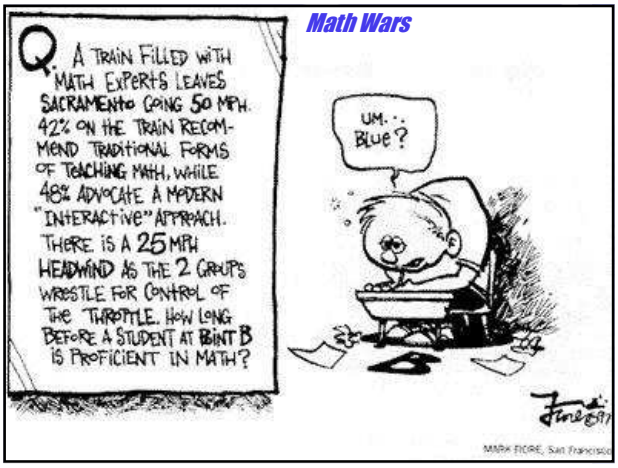
- Verbalizing and use of visuals for problem solving;
- Explicit instruction in how to use specific skills and multi-step strategies;
- Their teachers receiving feedback from formative assessment to modify instruction;
- Peer-assisted learning opportunities in which they focus on problem details, observe models of proficient students' problem solving, or are guided by more proficient peers

What is Mathematical Problem Solving?

- Problem solving is a central theme in the *Principles and Standards for School Mathematics* [National Council of Teachers of Mathematics (NCTM), 2000].
- Mathematical problem solving refers to "the cognitive process of figuring out how to solve a mathematics problem that one does not already know how to solve" (Mayer, 1996, p. 31).

Why Word Problems?

- Story problems represent "the most common form of problem solving" (Jonassen, 2003, p. 267) in school mathematics curricula.
- Story problems are more challenging than no context problems.



Traditional Models of Mathematical Problem Solving

I. Key word strategy

Associate key words (e.g., "more," "altogether," "gave away," "left over," "times," "among") with mathematical operations.

Examples:

Jill gave away 6 cookies in the morning. She gave away 2 cookies in the afternoon. How many cookies did she *give away* that day?" (Kelly & Carnine, 1996, p.5)

Jose´ took the 26 baseball cards he no longer wanted and gave them to Brian. Now Jose´ has 71 baseball cards *left*. How many baseball cards did Jose´ have to begin with?

Concerns with the Key Word Strategy

- Key words are misleading.
- Many problems have no key words.
- The key word strategy sends a terribly wrong message about doing mathematics

Van de Walle, 2004, p. 152

Traditional Models of Mathematical Problem Solving

II. Four-stage model (Polya, 1957)

- Understand the problem
- Develop a plan
- Carry out the plan
- Look back to check if the solution makes sense.

Van de Walle, 2004, p. 152

A Model of Mathematical Problem Solving

- Models for understanding and assessing children's solution of problems suggest that semantic structure is much more relevant than syntax in children's solutions of addition and subtraction problems (Carpenter, Hiebert & Moser, 1981; Carpenter & Moser, 1983).

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Classification of Word Problems

(Carpenter & Moser, 1983)

- Change: There is an initial quantity and a direct or implied action causes an increase or decrease in that quantity.
  - "Jose had 18 baseball cards. His friend Juan gave him 6 more baseball cards. How many baseball cards does Jose have now?"
- Combine or Group: Two distinct groups or subsets combine to form a new group or set.
  - "There are 12 students in a school play. 4 are boys and the rest are girls. How many girls are there?"
- Compare: The comparison of two distinct, disjoint sets (compared and referent).
  - "Andy's has 14 music CDs and Lisa has 9 music CDs. How many less CDs does Lisa have than Andy?"

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Mathematical Problem Solving

- Domain-specific knowledge (conceptual and procedural) is critical to successful mathematical problem solving (Hegarty, Mayer, & Monk, 1995).
  - Conceptual knowledge = Problem comprehension/representation
  - Procedural Knowledge = Basic numerical skills, arithmetic calculations.

Note. Procedural knowledge "is extremely limited unless it is connected to a conceptual knowledge base" (Prawat, 1989, p. 10).

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Schema-Based Instruction (SBI)

- SBI emphasizes the role of the mathematical structure (semantic) of a problem.
- A schema is an organized structure "consisting of certain elements and relations" specific to a situation (Mayer, 1999, p. 228).

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SBI

- Schemata are the appropriate mechanism for the problem solver to "capture both the patterns of relationships as well as their linkages to operations" (Marshall, 1995, p. 67).

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Schema-Based Problem Solving Model

- Schema knowledge/Problem Schema Identification
- Elaboration knowledge/Representation
- Strategic Knowledge/Planning
- Executive Knowledge/Solution

Marshall (1990); Mayer (1999); Riley, Greeno, & Heller (1983)

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Mathematical Problem Solving

Address the "big ideas," or "schemata" (e.g., change, group, compare) in teaching problem solving

Van de Walle (2004)

Word Problem Solving Curriculum: SBI

(Jitendra, 2007)

- Word problems selected from commonly adopted US mathematics textbooks and modified to meet the needs of students with diverse experiential backgrounds.
- Word problems formatted as verbal text, graphs, tables, and pictographs.
- Teaches "big ideas" or salient problem schemata (e.g., change, group, compare).
- Instruction focuses on both conceptual and procedural understanding - problem comprehension and problem solution.

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Word Problem Solving Curriculum: SBI

(Jitendra, 2007)

- Includes appropriate scaffolding of instruction.
  - Teacher-led instruction followed by paired learning and independent learning activities.
  - Tasks begins with story situations followed by word problems with unknown information.
  - Visual diagrams and checklists are used until students learn to apply the strategy independently.
- Incorporates adequate practice and mixed review of problem types.

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Word Problem Solving Curriculum: SBI

(Jitendra, 2007)

- Instruction is aligned with state assessment in terms of communicating, reasoning, and representing word problems.
- Employs frequent measures of student word problem solving performance to monitor student progress.

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Schema-Based Instruction (SBI): Application to Change Problems

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Phase 1: Schemata Identification and Representation Instruction

- Provides students with **story situations** of each problem type (e.g., change, group, compare) that do not contain any unknown information.
- Introduces **problem schema analysis** (i.e., discerning the key features of the problem) **using modeling** with several examples of story situations.
- During guided practice, **frequent student exchanges** are used to facilitate the identification of critical elements of the story.

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Phase 1: Problem Schemata Identification and Representation Instruction

- Uses a **self-monitoring checklist** to help students identify and map key information onto the diagram.
- Students learn to **underline** (e.g., words, sentences) **and circle** (e.g., numbers) **key information** in the problem before mapping it onto the schema diagram.

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WORD PROBLEM SOLVING STEPS (FOPS)

Step 1. Find the problem type.

Step 2. Organize the information in the problem using the diagram (change, group, or compare).

Step 3. Plan to solve the problem.

Step 4. Solve the problem.

CHANGE STORY CHECKLIST

Step 1. **F**ind **the** problem type.

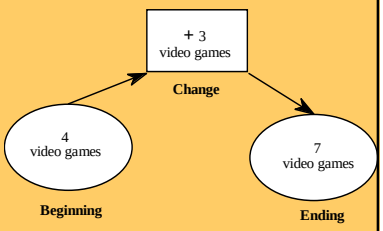
- ☐ Did I **read and retell** the story?
- ☐ Did I **ask if it is a change problem**? (Did I **look for the beginning, change, and ending?** Do they **all describe the same thing**?)

Step 2. **O**rganize **the** information **using the change diagram.**

- ☐ Did I **underline the label** that describes the beginning, change, and ending and **write the label** in the diagram?
- ☐ Did I **underline** important information, **circle numbers,** and **write numbers** in the diagram?

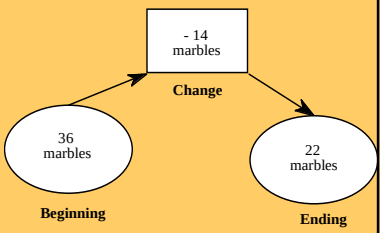
Example: Change Story Situation

Jane had 4 video games. Then her mother gave her 3 more video games for her birthday. Jane now has 7 video games.



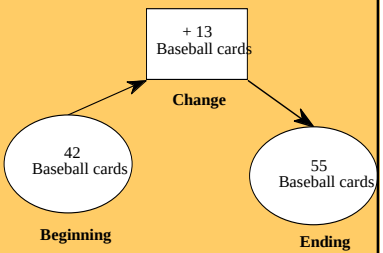
Example: Change Story Situation

Before he gave away 14 marbles, James had 36 marbles. Now he has 22 marbles.



Example: Change Story Situation

Tom had 42 baseball cards. He now has 55 baseball cards after he bought 13 more cards.



ADDITION AND SUBTRACTION ONE-STEP PROBLEMS: CHANGE

Problem Type	Example
Change	
Unknown ending amount (Addition)	Gail had 43 music albums in her collection. Then, she bought 11 albums at a garage sale. How many albums does Gail have now?
Unknown change amount (Subtraction)	Roger had 36 comic books. Then his father bought him some more for his birthday. Roger now has 52 comic books. How many comic books did he receive from his father?
Unknown beginning amount (Subtraction)	There were some Halloween masks in the fifth grade classroom. Then the class made 15 more masks. Now they have 42 masks. How many masks were in the classroom?

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

Phase II: Planning and Solution Instruction

- A 4-step (FOPS) procedure is used to solve all addition and subtraction problems.
- Encourages metacognition by having students use the problem checklist as they solve *change*, *group*, and *compare* problems.
- Diagrams are faded once students become independent in correctly mapping and solving the problem using the schematic diagram.

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WORD PROBLEM SOLVING STEPS (FOPS)

Step 1. Find the problem type.

Step 2. Organize the information in the problem using the diagram (change, group, or compare).

Step 3. Plan to solve the problem.

Step 4. Solve the problem.

CHANGE PROBLEM CHECKLIST

Step 1. Find the problem type.

- ☐ Did I **read and retell** the problem?
- ☐ Did I **ask if it is a change problem**? (Did I **look for the beginning, change, and ending?** Do they **all describe the same thing**?)

Step 2. Organize the information **using the change diagram.**

- ☐ Did I **underline the label** that describes the beginning, change, and ending and **write the label** in the diagram?
- ☐ Did I **underline** important information, **circle numbers,** and **write numbers** in the diagram?
- ☐ Did I **write a "?"** for what must be solved? (Did I **find the question sentence**?)

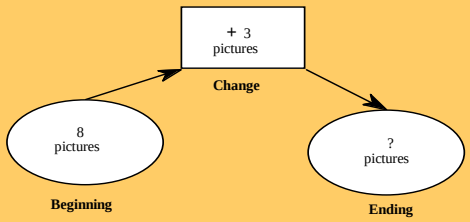
Step 3. Plan **to solve the problem.**

- ☐ Do I **add or subtract?** (If the **total or whole is given, subtract.**
- ☐ If the **total or whole is not given, add**)
- Did I **write the math sentence**?

Step 4. Solve **the problem.**

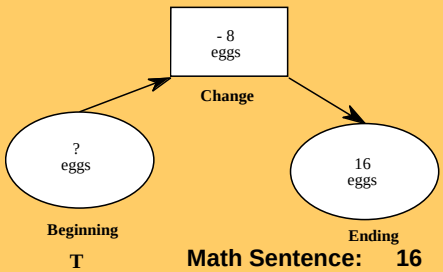
- ☐ Did I **solve** the math sentence?
- ☐ Did I **write the complete answer?**
- ☐ Did I **check if the answer makes sense?**

Change Problem: Tammy likes to paint pictures. She has painted 8 pictures so far. If she paints 3 more pictures, how many will she have?



Math Sentence: 8 T
+3

Change Problem: Tina bought some eggs. She used 8 eggs for breakfast. She now has 16 eggs. How many eggs did she begin with?



T

Math Sentence: 16
+ 8

Computational and Semantic Forms of Equations

Not always is the unknown quantity isolated on one side of the equal sign. For example, the likely equation for the earlier change problem is : ☐ - 8 = 16

This is referred to as the *semantic* equation for the problem since the numbers are listed in the order that follows the meaning of the problem.

When the semantic form does not isolate the unknown, an equivalent equation can be written for the same problem as follows: 16 + 8 = ☐

This equation is referred to as the *computational* form of the equation

Van de Walle. 2004. p. 138

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Using Symbols

Introduce symbols in the discussion portion of a lesson where students have solved story problems.
Say, "You had the two parts 8 and 16 in your problem. You found out that the whole number (beginning amount) you did not know was 24. Here is a way we can write that:

☐ - 8 = 16

The minus sign should be read as "minus" or "subtract" but not as "take away." The equal sign means "is the same as." Use this in place of or in conjunction with "equals" as you read equations with students.

Van de Walle. 2004. p. 138

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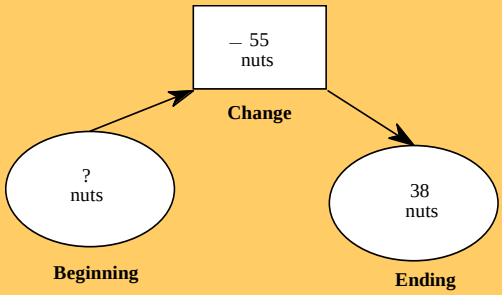
Change Answer Sheet: Paired Learning

The gray squirrel made a pile of nuts. It carried away 55 nuts up to its nest. Now there are 38 nuts left in the pile. How many nuts were in the pile at the beginning?

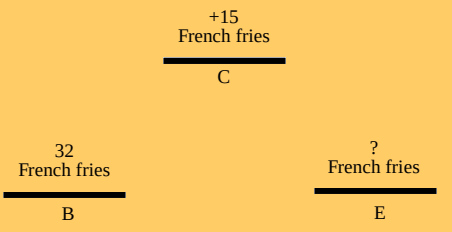


Change Answer Sheet (continued)

Work	Explanation
55 **+38** **93 nuts**	**First,** I figured this to be a change problem, because it has a beginning, a change, and an ending. **Next,** I used the change diagram to organize and represent the information in the problem. **Then,** I decided to add (55 and 38) to figure out the number of nuts in the pile at the beginning, because there were more nuts in the pile at the beginning than at the end. **Finally,** I wrote my math sentence and solved it. I also wrote the complete answer with the number and label.

Answer: 93 nuts

***Fading Change Diagrams*:** You had 32 French fries. You got 15 more French fries from your sister. How many French fries do you have now?



Math Sentence: 32
+15
47

SBI: Application to Group Problems

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GROUP STORY CHECKLIST

Step 1. **F**ind **the** problem type.

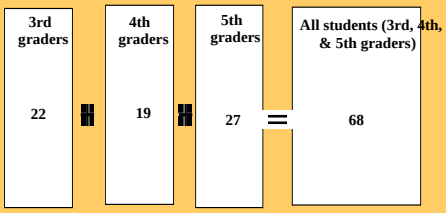
- ☐ Did I **read and retell** the story?
- ☐ Did I **ask if it is a group problem**? (Did I look to see if **two or more small groups combine to make up a large group**?)

Step 2. **O**rganize **the** information **using the group diagram.**

- ☐ Did I **underline the large group and small groups** and write **group names** in the diagram?
- ☐ Did I **circle numbers for the groups** and **write the numbers** for groups in the diagram**?**

Example: Group Story Situation

68 students at Hillcrest Elementary took part in the school play. There were 22 third graders, 19 fourth graders, and 27 fifth graders in the school play.



Small Groups or Parts | **Large Group or Whole (Sum)**

ADDITION AND SUBTRACTION ONE-STEP PROBLEMS: GROUP

Problem Type	Example
Group	
Unknown large group (whole) amount (Addition)	Meg saw 13 bear cubs running and 16 bear cubs walking at the zoo. How many bear cubs did Meg see at the zoo?
Unknown small group (part) amount (Subtraction)	In an apple picking contest, the third and fourth graders picked 84 apples. If the third graders picked 41 apples, how many apples did the fourth graders pick?

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

GROUP PROBLEM CHECKLIST

Step 1. **F**ind **the** problem type.

- ☐ Did I **read and retell** the problem?
- ☐ Did I **ask if it is a group problem**? (Do **two or more small groups combine to make up a large group**?)

Step 2. **O**rganize **the** information **using the group diagram.**

- ☐ Did I **underline the large group and small groups** and write **the group names** in the diagram?
- ☐ Did I **circle numbers for the groups** and **write numbers** for groups in the diagram**?**
- ☐ Did I **write a "?"** for what must be solved? (Did I **find the question sentence**?)

Step 3. **P**lan **to solve the problem.**

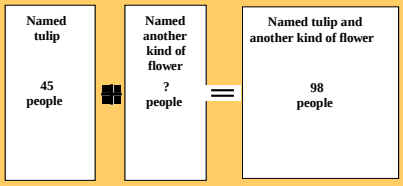
- ☐ Do I **add or subtract?** (If the **total or whole is given, subtract.** If the **total or whole is not given, add**)
- ☐ Did I **write the math sentence**?

Step 4. **S**olve **the problem.**

- ☐ Did I **solve** the math sentence?
- ☐ Did I **write the complete answer?**
- ☐ Did I **check if the answer makes sense?**

Group Problem

In a survey, 98 people were asked what their favorite flower is and 45 named tulips. How many named another kind of flower?



Small Groups or Parts | **Large Group or Whole (Sum)**

Math Sentence: 98

-45

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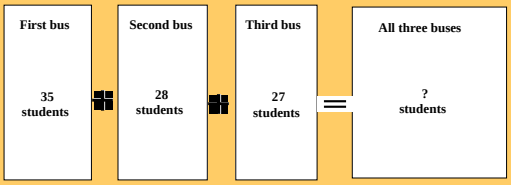
Group Answer Sheet: Paired Learning

Three buses took students on a field trip. One bus carried 35 students, another bus carried 28 students, and the third bus carried 27 students. How many students went on the trip?



Small Groups or Parts | **Large Group or Whole (Sum)**

Group Answer Sheet (continued)

Work	Explanation
35 28 +27 90 students	**First,** I figured this to be a group problem, because it has three small groups that combine to make a large group (all students). **Next,** I used the group diagram to organize and represent the information in the problem. **Then,** I decided to add 35, 28, and 27 to figure out all the students (large group) who went on the field trip, because the large group is the sum of the small groups. **Finally,** I wrote my math sentence and solved it. I also wrote the complete answer with the number and label.

Answer: 90 students went on the trip

***Fading Group Diagrams*:** A new bike costs $80. A new helmet costs $20. How much would it cost to buy the bike and the helmet?

Math Sentence: $80
+$20
$100

SBI: Application to Compare Problems

COMPARE STORY CHECKLIST

Step 1. **F**ind **the** problem type.

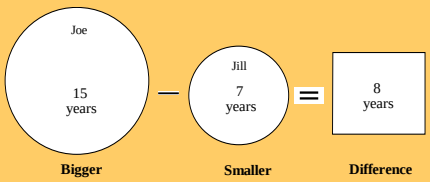
- ☐ Did I **read and retell** the story?
- ☐ Did I **ask if it is a compare problem**? (Did I look for compare words – ***taller than, shorter than, more than, less than?***)

Step 2. **O**rganize **the** information **using the compare diagram.**

- ☐ Did I **underline the comparison sentence** and **circle** the two **things compared**?
- ☐ Did I **reread the comparison sentence** to ask, "**Which is** the ***bigger*** amount and which is the ***smaller*** amount?" and **write the names of things compared** in the diagram?
- ☐ Did I **underline** important information, **circle numbers and labels** and **write numbers and labels** in the diagram**?**

Example: Compare Story Situation

Joe is 15 years old. He is 8 years older than Jill. Jill is 7 years old.



ADDITION AND SUBTRACTION ONE-STEP PROBLEMS: COMPARE

Problem Type	Example
Compare	
Unknown difference amount (Subtraction)	The pet store is having a sale of 21 hamsters and 32 kittens. How many more kittens are on sale than hamsters?
Unknown compared amount (Addition)	72 people came to the school play on Monday. 26 more people attended it on Tuesday than Monday. How many people went to the school play on Tuesday?
Unknown referent amount (Subtraction)	Janice is 85 centimeters tall. She is 16 centimeters taller than Melinda. How tall is Melinda?

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

COMPARE PROBLEM CHECKLIST

Step 1. **F**ind **the** problem type.

- ☐ Did I **read and retell** the problem?
- ☐ Did I **ask if it is a compare problem**? (Did I look for compare words – ***taller than, shorter than, more than, less than*?)**

Step 2. **O**rganize **the** information **using the compare diagram.**

- ☐ Did I **underline the comparison sentence or question** and **circle** the two **things compared**?
- ☐ Did I **reread the comparison sentence or question** to ask, "**Which is** the ***bigger*** amount and which is the ***smaller*** amount?" and **write the names of things compared** in the diagram?
- ☐ Did I **underline** important information, **circle numbers and labels** and **write numbers and labels** in the diagram?
- ☐ Did I **write a "?"** for what must be solved? (Did I **find the question sentence**?)

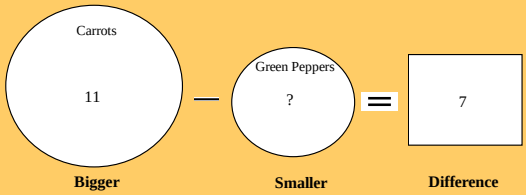
Step 3. **P**lan **to solve the problem.**

- ☐ Do I **add or subtract?** (If the **total or whole is given, subtract.** If the **total or whole is not given, add**)
- ☐ Did I **write the math sentence**?

Step 4. **S**olve **the problem.**

- ☐ Did I **solve** the math sentence?
- ☐ Did I **write the complete answer?**
- ☐ Did I **check if the answer makes sense?**

Compare Problem: Steve picked 11 carrots. He picked 7 fewer green peppers than carrots. How many green peppers did Steve pick?

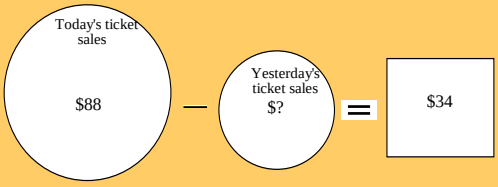


Bigger **Smaller** **Difference**

Math Sentence: 11
- 7
4

Compare Answer Sheet: Paired Learning

Today ticket sales were $88 for the circus. This is $34 more than the total sale of yesterday. How much money was collected yesterday?



Bigger **Smaller** **Difference**

Compare Answer Sheet (continued)

Work	Explanation
$88 **-$34** **$54**	**First**, I figured this to be a compare problem, because it is comparing yesterday's ticket sales to today's ticket sales. **Next**, I used the compare diagram to organize and represent the information in the problem. **Then,** I decided to subtract $34 from $88 to figure out the amount for yesterday's ticket sales, which is less than today's ticket sales. **Finally,** I wrote my math sentence and solved it. I also wrote the complete answer with the dollar sign and number.

Answer: $54

***Fading Compare Diagrams*:** Lin is 5 years older than his cousin. If Lin is 11 years old, how old is his cousin?

Lin		Lin's cousin		
11 years	—	? years	=	5 years
B		S		D

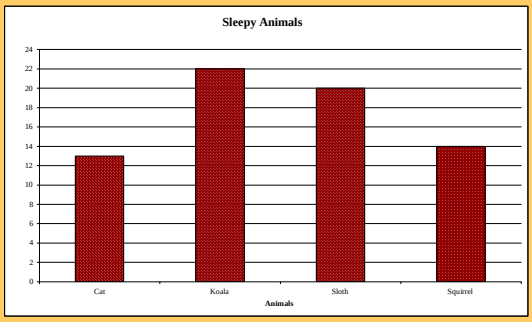
Math Sentence: 11
- 5
6

Mixed Review:One-Step Problems

1. In one week Samuel read 35 pages. He read 16 fewer pages than Wes. How many pages did Wes read?
2. Ted collected some pictures of butterflies in his scrapbook. This week, he added 25 more pictures. Now he has 90 pictures of butterflies in his scrapbook. How many pictures did he have in the beginning?
3. At top speed, a giraffe can run 32 miles an hour. This speed is 3 miles an hour faster than that of an antelope. How many miles an hour does the antelope run?
4. Your teacher made some snacks for the class. There were 8 left after the students ate 14 snacks. How many snacks did the teacher make for the class?
5. You have a collection of 12 marbles. If 5 of the marbles in your collection are large, how many marbles are small?
6. Karen had 16 of her friends come to her birthday party. 6 of her friends left the party early. How many are still at Karen's birthday party?
7. Olivia has two puzzles. A balloon picture puzzle has 25 pieces. A boat picture puzzle has 5 fewer pieces than the balloon picture puzzle. How many pieces does the boat picture puzzle have?

Mixed Review:One-Step Problems

Use the graph to solve the problem below.

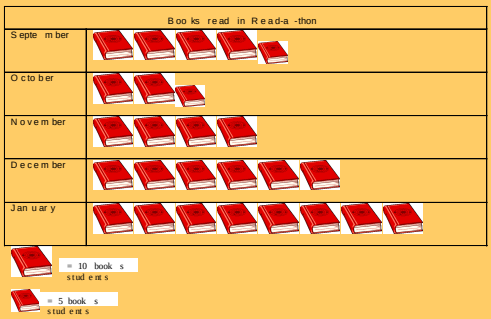


8. How many more hours does the koala sleep than the squirrel?

Problems Illustrated as Pictographs, Tables, and Graphs

Word Problem: Pictograph

Use data from the pictograph to solve Problem 1.



1. If third graders in Mrs. Blake's class started reading in September and read some memorable books in October. How many books did they read at the end of October?

Word Problem: Table

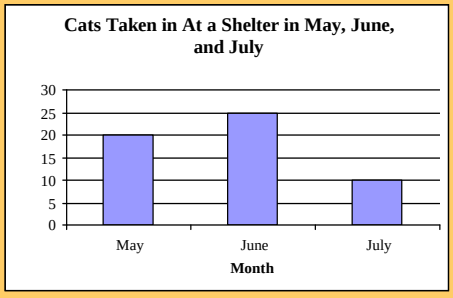
Use the Table below to solve Problem 2.

Third Grade Students at County Elementary School	
Ms. Griffin's classroom	20
Mrs. Smith's classroom	19
Mr. Chard's classroom	18
Ms. Howard's classroom	16

2. Suppose 5 new students will be entering Ms. Howard's class next year and none of the current students leave. How many students will there be in Ms. Howard's class?

Word Problem: Graph

Use the information in the graph below to solve problem 3.



3. How many cats did the shelter take in during May, June, and July?

Solving Two-Step Addition and Subtraction Problems

Solving Two-Step Addition and Subtraction Problems

Students learn to identify the:

- Overall or primary problem schema to be solved and write PA or partial answer for the missing element in the primary schema diagram that could be found by solving the the secondary problem.
- Secondary problem that must be solved to answer the primary problem

TWO-STEP PROBLEM CHECKLIST

Step 1. **Ask** if the problem is a two-step problem.

- ☐ Did I **read and retell the problem to figure out if it is a two-step problem?** (Does the **problem involve more than one step or problem type**?)

Step 2. **Plan and organize** to solve the two-step problem.

- ☐ Did I **find the primary problem** to be solved?
- ☐ Did I **organize the information in the primary problem** using a diagram (change, group, or compare)?
- ☐ Did I **find the secondary problem** to be solved?
- ☐ Did I **organize the information in the secondary problem** using a diagram (change, group, or compare)?

Step 3. **Solve** the problem.

- ☐ Did I **solve for the unknown in the secondary problem**?
- ☐ Did I **solve for the unknown in the primary problem?**
- ☐ Did I **write the complete answer**?
- ☐ Did I **check if the answer makes sense**?

Example: Two Step Problem

Use the data in the table below to solve this problem.

Bob's puppy and weight gained

Month	Weight
September	6 pounds
October	4 pounds
November	5 pounds

Bob's puppy gained weight from September to November. How many more pounds did the puppy gain in September and October than in November?

Two-Step Problem

Primary Problem

Sept. & Oct. 10 ~~PA~~ pounds — **Nov.** 5 pounds = 5 ~~?~~ pounds

B **S** **D**

10 - 5 = 5

Secondary Problem

Sept. 6 pounds + **Oct** 4 pounds = **Sept. & Oct.** 10 PA ~~?~~ pounds

SG **SG** **LG**

6 + 4 = 10

Example: Two-Step Problem

- Amy goes up 7 steps and then back down 3 steps to pick up a book she dropped. Then she goes up 5 steps. How many steps did she go up in the end?

Secondary Problem

B: 7 steps — C: - 3 steps — E: 4 PA ~~steps~~

7 - 3 = 4

Primary Problem

B: 4 ~~PA~~ steps — C: + 5 steps — E: 9 ~~steps~~

4 + 5 = 9

Example: Two-Step Problem

- Keith is waiting in line to buy snacks. There are 8 people ahead of him. Two people leave the line without buying anything. Two people buy their snacks and go to their seats. How many people are ahead of him now?

Secondary Problem

B: 8 people — C: - 2 people — E: 6 PA ~~?~~ people

8 - 2 = 6

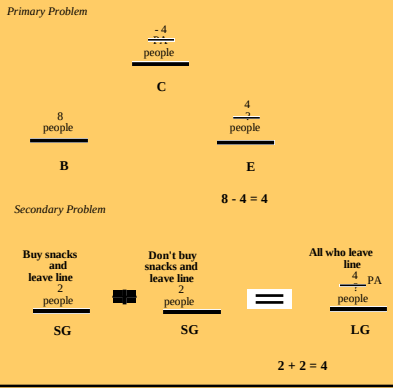
Primary Problem

B: 6 ~~PA~~ people — C: - 2 people — E: ? people

6 - 2 = 4

Two-Step Problem (Alternate solution)

- Keith is waiting in line to buy snacks. There are 8 people ahead of him. Two people leave the line without buying anything. Two people buy their snacks and go to their seats. How many people are ahead of him now?



Other Factors to Consider when Teaching Problem Solving

Guidelines for Successful Paired Learning

- **Assign partners before the assignment**. Partners should be heterogeneously grouped.
- **Change partners**.
- **Monitor discussions.** Circulate around the classroom and monitor student work to assess student comprehension of the task.

Monitoring Problem Solving Performance

- Monitor student competence and growth on mathematical word problem solving skill on an on-going basis.
- At the end of each instructional session, check students' understanding and evaluate whether they mastered the strategy steps.
- Examine students' completed worksheets for strategy use (e.g., drawing a diagram, mapping information onto the diagram, writing the math sentence) and provide them with additional instruction as needed before they move to the next problem type.

Summary: Guidelines to Consider

- Introduce schema strategy instruction by providing **explicit modeling and explanations** using several examples.
- Use **frequent student exchanges** to check student understanding of strategy steps and provide corrective feedback.
- Provide **scaffolded instruction** (e.g., model mapping, present schemata diagrams and checklists) as students learn to apply the strategy and **eventually fade** the diagrams.
- **Monitor student use of the strategy steps** during guided, paired, and independent learning activities.

Findings from our Research on SBI: Addition and Subtraction Problems

- Seven studies investigated the effects of SBI on the problem solving skills of students with disabilities (Griffin & Jitendra, in press; Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998; Jitendra & Hoff, 1993; Jitendra, Hoff, & Beck, 1999; Jitendra, Grasso, & DiPipi, 2001; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007). Overall, results indicated that:
  - Students improved problem solving scores immediately following the intervention.
  - Strategy effects were maintained for as long as 2 to 6 weeks after the termination of the study.
  - Students generalized the learned skill to novel (e.g., multi-step) problems as well as to problems on the PSSA mathematics test.

Findings from our Research on SBI: Addition and Subtraction Problems

Other Findings

- Following SBI, students' performance either approached or surpassed that of a normative sample.
  - Scores on the immediate posttest (mean correct =77%) and delayed posttest (81%) for the schema group approached those of a normative sample of third graders (posttest *M* = 81%; delayed posttest *M* = 82%) (Jitendra, Grifffin, McGoey, Gardill, Bhat, & Riley, 1998).
  - Students' performance on two-step addition/subtraction word problems (*M* = 86% correct) surpassed that of the normative sample (*M* = 54% correct) (Jitendra, Hofff, & Beck, 1999).

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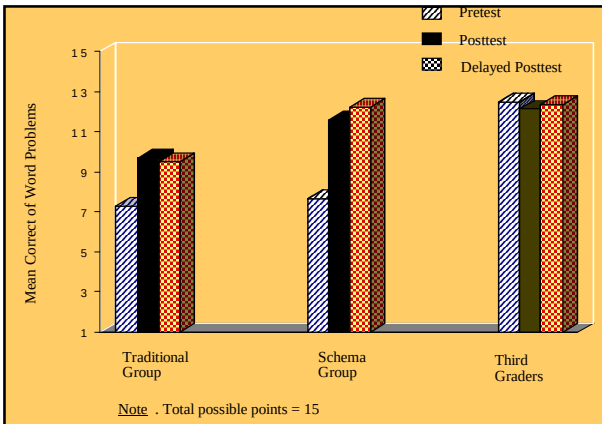

Note . Total possible points = 15

Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

Instructional Phase	Students with Learning Disabilities	Third Graders without Disabilities
Baseline		
One-step problems	55%	49%
Two-step problems	15%	26%
Post-Instruction on One-step problems		
One-step problems	72%	53%
Two-step problems	67%	55%
Post-Instruction on Two-step problems		
One-step problems	84%	85%
Two-step problems	86%	54%

Jitendra, Hoff, & Beck (1999)

General Findings from our Research on SBI

- ***Strategy Use.*** We examined worksheets to determine the extent to which students effectively applied SBI to solve word problems. In general, students were more likely to engage in strategy use (e.g., draw diagrams, map diagrams, write the total, write number sentences) following instruction than before SBI.

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General Findings from our Research on SBI

Students reported positive attitudes regarding the strategy instruction.

For example,

- Students' responses about what they liked the most about the strategy were as follows: "learning to solve problems," "getting the correct answers," "it's easier to understand, it gave me clues," "I liked the problems."
- When asked if they would recommend this strategy to a friend, they answered in the positive.

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Strategy Satisfaction: Students with LD and LA Students

Strategy Questionnaire		LD (n = 9)	LA (n = 9)	Total	ES
Enjoyed (5/5)	M	4.56	3.44	4.00*	+1.20
	SD	0.73	1.13	1.08	
Diagram (5/5)	M	4.44	4.00	4.22	+0.44
	SD	1.13	0.87	1.00	
Help solve (5/5)	M	4.89	3.44	4.17***	+2.16
	SD	0.33	1.01	1.04	
Recommend (5/5)	M	4.56	3.89	4.22	+0.65
	SD	1.01	1.05	1.06	
Continue (5/5)	M	4.44	3.00	3.72**	+1.40
	SD	0.73	1.32	1.27	
Total (5/5)	M	22.89	17.78	20.33**	+1.71
	SD	3.26	2.73	3.93	

Jitendra, Deatline-Buchman, & Sczesniak (2007)

General Findings from our Research on SBI

- Teachers of students in our projects indicated transfer of effective problem solving skills into their classrooms.
  - One teacher noted that the student who evidenced math phobia prior to the SBI showed increased confidence in her math performance and was more consistent in labeling her work on problem-solving tasks.

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